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Abstract

How does international trade affect income redistribution? We consider a country where the government uses a nonlinear income tax to maximize some redistributive social welfare function, subject to the constraint that it can observe only individual income but not individual characteristics. In autarky, the government is able to partially equalize equilibrium prices and wages by manipulating quantities through the tax system. If borders are open, prices and wages are determined by world markets and the government is deprived of this possibility. This implies that it may be unable to redistribute the gains from trade: opening borders may decrease welfare even after the optimal policy adjustment.

Keywords: gains from trade, redistribution, income taxation
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1 Introduction

Assume that a country previously in autarky opens up to international trade. This yields an efficiency gain, as the set of feasible allocations expands. At the same time, trade causes a change in relative prices and wages, inducing distributive effects. If the government were perfectly informed, it would be able to transfer income from the gainers to the losers, so as to make opening up to trade Pareto improving. For example, if different types of agents (say, skilled and unskilled) produce different goods, and trade affects their relative prices, then the government could transfer income from workers whose skills became more valuable to those whose skills became less valuable. However, the government is not likely to possess enough information about individual skills to follow such a policy. It is more realistic to assume that it is only able to observe individuals' total incomes, and is constrained to use income taxation. Incentive reasons constrain the amount of possible redistribution, and a question of paramount importance for the political economy of trade is whether, in spite of these constraints, it is nonetheless possible to make trade Pareto improving. For the question to make sense, we compare a closed economy and an open economy, assuming that in both cases the government maximizes the same welfare function, using an income tax at the exclusion of any other instrument. We analyze how welfare is affected by the opening of borders.

We show that the government may be unable to make sure that opening up to trade increases welfare. The mechanism at work is driven by a qualitative change in the set of policies available to the government. Fiscal redistribution in a closed economy can be described as operating through two channels: on the one hand, the government transfers income from the rich to the poor. But it also affects the pre-transfer equilibrium, through the effect of taxes on labor supply (or quantities more generally). If, as is assumed for simplicity in most of the optimal taxation literature (following Mirrlees [1971]), prices and wages are exogenous, then the equilibrium change is only a change in quantities, not in prices, and it is harmful in itself: were it not accompanied by income transfers, the distortion of quantities would decrease welfare. In other words, the equilibrium change is the cost to pay to allow for transfers. However, if prices are endogenous, the government seeks to affect them through the tax scheme, and it can redistribute by causing wage inequality to decrease (see Stiglitz [1982]). The reason - at the core of the

point made in this paper - is that the constraints on redistribution make it impossible to entirely offset changes in relative wages, so that the optimal scheme also tries to directly increase low wages. In such an environment, it is possible, at least in theory, that the welfare gain of the poor be mostly derived from the change in equilibrium wages, and only secondarily from transfers[1].

When a country opens up to trade, the elasticity of the demand for its goods, and hence for the different types of labor, increases. In the extreme case where the country is small and all goods are tradable, it becomes infinite, since prices are determined by world markets. It becomes impossible therefore for the government to affect equilibrium wages, and one of the two redistributive tools available in a closed economy disappears. This can make it impossible to maintain everyone's welfare at the level reached before the opening of borders.

This mechanism is very different from the one that would prevail if there were no government. In the latter case, the poor would simply be hurt if the world at large valued their skills less than the country in isolation, and the greater the difference, the greater their welfare loss. If there is a government, on the contrary, we show that the effect of trade on welfare, as a function of world prices, is U-shaped: if world prices are very different from autarky prices, the gains from trade are large enough to offset any harmful effect: a small redistribution is enough to make everyone better off. Conversely, if identical countries form a common market (which would have no consequence if there were no government), then welfare decreases: each government faces an increase in the demand elasticities, and it becomes impossible to affect equilibrium wages.

The paper most closely related to ours is Naito [1998]. Building on the idea that the constraints on fiscal redistribution leave a role for policies directly affecting wages, he proved an important result about trade and redistribution: if the government has redistributive preferences, and can use a nonlinear income tax as well as tariffs, then it enacts tariffs (or export subsidies) at the optimum[2]. The logic is simple: if there are initially no tariffs, then the adverse effect of a small tariff, due to resource misallocation, is

[1] See Zeckhauser [1977] for a very amusing illustration.

[2] Dixit and Norman [1980, 1986] show that if the government can tax income and subsidize some goods, it should not use tariffs. Naito [1998] proves that this well-known result breaks down if the type of labor supplied by an individual is not observable by the government.

second-order, while the effect on equilibrium wage inequality, and hence on social welfare, is first-order.

In the light of this result, our question (is it at least possible, using income taxation but no tariffs, to redistribute the gains from trade in a Pareto improving manner?) may seem unimportant: Naito [1998] tells us that the optimal scheme should make use of tariffs! But there are good reasons to beware of tariffs: like commodity taxes, they make it very easy for politicians to cater to special interests, and they lend themselves to wasteful rent-seeking to a larger extent than income taxation. Therefore, although they may be part of an optimum, they might be dismissed on the grounds of a skeptical view of the political process[3]. This dismissal however would be weakened if it turned out that income taxation alone may not be sufficient to redistribute the gains from trade.

The paper is organized as follows: after presenting the general model in Section 2, we solve the optimal taxation problem in an open economy (Section 3) and in an closed economy (Section 4). Comparative statics are then studied in Section 5, where the main results are stated. Two examples clarify these results in Section 6, and we conclude in Section 7.

2 The model

We develop here a very simple model of optimal taxation. It will boil down to a particular case of Mirrlees [1971] if the economy is open and wages are exogenous (Section 3), and to a particular case of Stiglitz [1982] if the economy is closed and wages are endogenous (Section 4). Notice that, while the distinction between direct transfers and indirect redistribution through wage manipulation, made in the introduction, may be relevant to think about the real world, it is not explicitly modeled: the only policy variable is a tax scheme (equivalently modeled as the choice of a pair of labor supplies and of an income transfer).

We consider a country inhabited by agents who can be of two types, skilled or unskilled, in respective proportions π_s and π_u ($\pi_u + \pi_s = 1$)

[3]Rodrik [1985] surveys the literature on the political economy of tariff formation. A possible way of alleviating the rent-seeking problem is to constrain the outcome by imposing that tariffs be uniform across goods. However, this is economically undesirable, because tariffs enacted for redistributive purposes should treat different goods differently, according to the mix of skills used in their production.

There are two consumption goods, labeled s and u (for skilled and unskilled). Each good is produced from the corresponding type of labor under a constant returns to scale production function: one unit of skilled (resp. unskilled) labor produces one unit of skilled (resp. unskilled) good:

$$\begin{cases} X_s = L_s \\ X_u = L_u \end{cases}$$

where X_s and X_u denote, respectively, the output of skilled and unskilled good, and L_u (resp. L_s) denotes the quantity of unskilled (resp. skilled) labor[4]. This implies that the price of good i is equal to the wage of type i labor, denoted w_i throughout the paper.

All agents have the same preferences, characterized by the utility function

$$U(C_s, C_u, l) = F(C_s, C_u) - V(l) \tag{1}$$

where C_i and l respectively denote individual consumption of good i and labor supply. F is assumed to be strictly concave, infinitely differentiable, increasing in each argument and homogeneous of degree one[5]. V is infinitely differentiable, strictly increasing, strictly convex and such that $V'(0) = 0$, $V'(\infty) = \infty$, and

$$l \to lV'(l) \text{ is a strictly convex function.} \tag{2}$$

Assumption (2) ensures that the optimal nonlinear taxation problems considered below are well-behaved[6].

That F is homogeneous of degree one has the following implications:

- The ratio of marginal utilities at a given consumption bundle is only a function of the ratio of quantities consumed. Formally, there exists a strictly increasing and infinitely differentiable function w such that for any pair of strictly positive consumptions (C_s, C_u),

$$\frac{\partial F}{\partial C_u} / \frac{\partial F}{\partial C_s} (C_s, C_u) = \frac{\partial F}{\partial C_u} / \frac{\partial F}{\partial C_s} \left(\frac{C_s}{C_u}, 1 \right) = w \left(\frac{C_s}{C_u} \right)$$

[4]This production function, allowing for no substitution at all in production, is extreme, but it allows to keep the model simple enough. We argue in the conclusion that our results are robust to relaxing it.

[5]Homogeneity of degree one simplifies calculations but is by no means essential to get to our results. The assumption of strict concavity will be relaxed in Section 6.2.

[6]Assumption 2 is identical to Assumption (45) in Mirrlees [1971] (p. 186).

We assume that the marginal utility of a good tends to zero as its quantity tends to infinity, the quantity of the other good remaining constant[7]. In other words:

$$\mathop{Lim}_{x \to 0} w(x) = 0 \text{ and } \mathop{Lim}_{x \to \infty} w(x) = \infty. \tag{3}$$

- (3) implies that whatever prices are, it is optimal to consume a strictly positive amount of each good (given some strictly positive income). We define the function x by the equation

  $$w(x(w)) = w \text{ for any } w > 0.$$

  $x(w)$ is equal to the optimal consumption ratio when the price ratio is w. Then, if we normalize any pair of prices (w_s, w_u) by imposing the equality

  $$\frac{\partial F}{\partial C_i}(x(w_u/w_s), 1) = w_i \ (i = u, s), \tag{4}$$

  the indirect utility achieved by an agent whose income and labor supply are respectively Y and l is

  $$Y - V(l).$$

We will therefore normalize prices using (4). This allows to define the functions $w_u(w)$ and $w_s(w)$ by the equations

$$w_i(w) = \frac{\partial F}{\partial C_i}(x(w), 1) \ \ (i = u, s). \tag{5}$$

$(w_u(w), w_s(w))$ is the only pair of prices satisfying (4) and such that $w_u(w)/w_s(w) = w$. Clearly, the fact that $x(w)$ is a strictly increasing function implies that $w_u(w)$ and $w_s(w)$ are, respectively, strictly increasing and strictly decreasing with w.

[7]This assumption simplifies the presentation and can be relaxed without affecting any result.

3 Optimal taxation in an open economy

In a small open economy with neither tariffs nor commodity taxes, prices are exogenously determined by the world market. Since one unit of a given type of labor produces one unit of the corresponding good, wages are exogenous as well[8]. The outside environment can be characterized by the price ratio $w = p_u/p_s$, which is also the wage ratio in the country. If we impose the normalization (4), the wages of the unskilled and of the skilled are respectively $w_u(w)$ and $w_s(w)$ as given by equation (5). We restrict our attention to the case where $w \leq 1$.

We assume throughout the paper that the government's objective is to maximize the social welfare function S given by

$$S = U_u,$$

where U_u is the unskilled workers' utility level. If the unskilled wage is lower than the skilled wage, as we are going to assume thoughout the paper, this social welfare function coincides with a rawlsian objective function[9]. The government is restricted to using a nonlinear income tax, at the exclusion of any other instrument, such as commodity taxes[10]. It is only able to observe an individual's total income, but not his type or, which would be equivalent, his hourly wage or his labor supply. This is the standard assumption of the optimal taxation literature: the informational asymmetry between individuals and the government imposes limits on redistribution.

The government's problem can be formulated as the choice of individual labor supplies l_u and l_s, and of a net transfer t paid by each skilled worker, so that each unskilled worker receives $t\pi_s/\pi_u$,.subject to the following incentive-compatibility constraint: given l_u and l_s, and the corresponding pre-tax incomes y_u and y_s, skilled workers should prefer (at least weakly) to earn y_s (which requires to work l_s hours) and pay t, rather than to earn y_u (which requires $l_u w_u/w_s = wl_u$ hours) and receive the transfer $t\pi_s/\pi_u$, or

$$y_s - t - V(l_s) \geq y_u + t\frac{\pi_s}{\pi_u} - V(wl_u).$$

[8]This statement relies on our assumption that production technologies allow for no substitution at all. See the conclusion for a discussion about relaxing this assumption.

[9]All our results would hold if the social welfare function were $\lambda U_u + (1-\lambda)U_s$ with $\lambda > \frac{1}{2}$.

[10]In this model, allowing for commodity taxes would amount to assuming that the government can observe individual types.

The maximal transfer is therefore such that this inequality is an equality, or

$$t = \pi_u \left(y_s - y_u - \left(V(l_s) - V(wl_u) \right) \right).$$

The government's objective function implies that this equality holds at the optimum, so that social welfare can be written as a function of l_u and l_s as follows:

$$SW_{open}(l_s, l_u, w) = y_u + t\frac{\pi_s}{\pi_u} - V(l_u) = y - \pi_s \left(V(l_s) - V(wl_u) \right) - V(l_u), \tag{6}$$

where $y = \pi_u w_u(w) l_u + \pi_s w_s(w) l_s$ is the country's average income.

This is the "classical" income taxation problem studied by Mirrlees [1971]. The following lemma summarizes some properties of its solution which will matter in the comparative statics analysis of Section 5.

Lemma 1 *The optimization problem (6) has a unique solution $(l_u(w), l_s(w))$. The functions $l_s(w)$ and $l_u(w)$ are continuous, and respectively strictly decreasing and strictly increasing.*

Proof. See the appendix. ∎

4 Optimal taxation in a closed economy

We consider now the same country in autarky. The incentive constraints on redistribution are the same, so that the government's goal is still to maximize the expression given by (6) above. The only difference with the open economy is that the wage ratio w is not exogenous any more. Rather, it is equal to the ratio of marginal utilities induced by the relative quantities produced (and therefore consumed) in the country, or $w = w\left(\pi_s l_s / \pi_u l_u\right)$. This allows to rewrite the government's objective function as

$$\begin{aligned} SW_{closed}(l_s, l_u) &= SW_{open}(l_s, l_u, w(\pi_s l_s / \pi_u l_u)) \\ &= F\left(\pi_s l_s, \pi_u l_u\right) - \pi_s \left(V(l_s) - V(l_u w(\pi_s l_s / \pi_u l_u)) \right) - V(l_u) \end{aligned} \tag{7}$$

(the term $F\left(\pi_s l_s, \pi_u l_u\right)$ results from the identity $y = F\left(\pi_s l_s, \pi_u l_u\right)$, which holds because F is homogeneous of degree one). We assume that the maxi-

mization of (7) has a unique[11] solution $\left(\hat{l}_s, \hat{l}_u\right)$. Let $\hat{w}$ and $\hat{S}$ denote respectively the wage ratio and social welfare (equal to the utility of the unskilled) at this optimum. We assume that $\hat{w} < 1$: the unskilled wage is lower than the skilled wage.

Comparing the problems (7) and (6) reveals that the only difference between an open and a closed economy is that in a closed economy the government takes into account the effect of quantities of relative wages. If it wants to redistribute towards the unskilled, it tries to increase the ratio w, which pushes the optimal skilled labor supply upwards and the optimal unskilled labor supply downwards, relative to the open economy optimum. As Stiglitz [1982] (of which this section is a particular case) notices, this implies that, at the optimum, skilled workers face an implicit negative marginal tax rate (unlike the zero marginal rate they face in an open economy), and unskilled workers a positive marginal rate, greater than the (already positive) one they would face in an open economy.

5 Comparative statics

In this section we investigate how social welfare varies with the world price ratio w, and we compare welfare in an open economy and in a closed economy, assuming that in both cases the government implements the welfare-maximizing tax scheme. We define social welfare at the optimum by $S(w) = SW_{open}(l_s(w), l_u(w), w)$. We are interested in how $S(w)$ varies with w, and in comparing the values reached by the function S with the welfare level $\hat{S} = S_{closed}(\hat{l}_s, \hat{l}_u)$ attained in autarky. We start with the case where w is equal to $\hat{w}$, the wage ratio prevailing in autarky.

Proposition 2 $S(\hat{w}) > \hat{S}$: *if the world prices are the autarky prices, then opening up to trade increases welfare. Also, the country exports the unskilled good (and imports the skilled good).*

Proof. The welfare level in autarky is obviously attainable if the world price ratio is $\hat{w}$, simply by assigning the same pair of labor supplies $\left(\hat{l}_s, \hat{l}_u\right)$ as

[11]The assumptions about V ensure that the problem (7) has at least one solution, which satisfies the two first-order conditions. Generically, the solution is unique: for two pairs of labor supplies (i.e., four variables) to solve (7), they must satisfy five equations: the two pairs of first-order conditions and the equation stating that the value of the maximand is the same at both pairs. This is generically impossible.

in autarky. This implies that $S(\hat{w}) \geq \hat{S}$. We show now that $(l_u(\hat{w}), l_s(\hat{w})) \neq (\hat{l}_s, \hat{l}_u)$. Differentiating (7) and (6) with respect to l_i ($i = u$ or s) leads to

$$\begin{aligned}\frac{\partial}{\partial l_i}(SW_{open})(\hat{l}_s, \hat{l}_u, \hat{w}) &= \frac{\partial}{\partial l_i}(SW_{closed})(\hat{l}_s, \hat{l}_u) - \frac{\partial}{\partial w}(SW_{open})(\hat{l}_s, \hat{l}_u, \hat{w})\frac{\partial}{\partial l_i}\left(w(\tfrac{\pi_s l_s}{\pi_u l_u})\right) \\ &= -\frac{\partial}{\partial w}(SW_{open})\frac{\partial}{\partial l_i}\left(w(\pi_s l_s/\pi_u l_u)\right).\end{aligned}$$

This and the inequality $\frac{\partial}{\partial w}(SW_{open}) > 0$ imply that

$$\begin{cases} \frac{\partial}{\partial l_u}(SW_{open})(\hat{l}_s, \hat{l}_u, \hat{w}) > 0 \\ \frac{\partial}{\partial l_s}(SW_{open})(\hat{l}_s, \hat{l}_u, \hat{w}) < 0. \end{cases}$$

We showed in the proof of Lemma 1 (in the appendix), that SW_{open} is the sum of a strictly concave function of l_s and of a strictly concave function of l_u. Therefore the inequalities above imply that $l_u(\hat{w}) > \hat{l}_u$ and $l_s(\hat{w}) < \hat{l}_s$, so that $(l_u(\hat{w}), l_s(\hat{w})) \neq (\hat{l}_s, \hat{l}_u)$ and

$$S(\hat{w}) = S_{open}\left(l_u(\hat{w}), l_s(\hat{w}), \hat{w}\right) > S_{open}(\hat{l}_s, \hat{l}_u, \hat{w}) = S_{closed}(\hat{l}_s, \hat{l}_u) = \hat{S}.$$

Finally, when the price ratio is $\hat{w}$, the optimal consumption ratio of skilled versus unskilled good is the same as in a closed economy: $x(\hat{w}) = \pi_s \hat{l}_s / \pi_u \hat{l}_u$, which is greater than the production ratio $\pi_s l_s(\hat{w}) / \pi_u l_u(\hat{w})$. This implies that the country imports the skilled good and exports the unskilled good. ■

This result may seem surprising at first: if there were no government, and world prices were the same as domestic prices, then opening up to trade would make no difference. The reason why this is not true any more if there is a government is that the tax scheme in autarky distorts labor supply decisions in order to compress the wage distribution: these distortions are the cost paid by the country to reach the relatively "egalitarian" wage ratio $\hat{w}$. By opening up to trade (assuming the world price ratio is $\hat{w}$), a country manages to maintain this wage ratio while removing the distortions[12], which increases welfare. Skilled workers, who were induced to work "too much" in autarky in order to increase the unskilled wage, are now working less, and, symmetrically, unskilled workers are working more. This implies that, after borders are opened, the country has an excess production of unskilled

[12] More precisely, the distortion of skilled labor supply is removed, and it is reduced for unskilled labor supply (it remains distorted downwards even when wages are exogenous, as in Mirrlees [1971]).

good, and exports it. The welfare gain can be understood by viewing the autarky price ratio $\hat{w}$ as high: it is greater than the wage ratio in the autarkic competitive equilibrium without any redistribution. Therefore opening up to trade in a world where $\hat{w}$ is the prevailing wage ratio amounts to opening up to a world where there is an inelastic demand for unskilled labor at a relatively high wage.

To continue the analysis of the function $S(w)$, we must introduce a few notations. For every w, we write respectively $x_u(w)$ and $x_s(w)$ for the per capita consumption of unskilled and skilled good, respectively, and $t_u(w)$ for the net export of unskilled good per capita, defined by $t_u(w) = \pi_u l_u(w) - x_u(w)$. The following lemma holds:

Lemma 3 $S'(w) = t_u(w)w_s(w) + \pi_s l_u(w)V'(l_u w)$.

Proof. See the appendix. ■

This result has an intuitive interpretation: the term $t_u(w)w_s(w)$ captures a "terms of trade" effect: if the country exports (resp. imports) the unskilled good, then an increase in its price improves (resp. worsens) the terms of trade. The second term, $\pi_s l_u(w)V'(l_u w)$, reflects the change in wages, and is always positive, since an increase in the unskilled wage increases the utility of the unskilled even after redistribution. We can now state a result comparing welfare in autarky with welfare in an open economy in the particular case where there is no trade in equilibrium.

Proposition 4 *There exists $w^* < \hat{w}$ such that $t_u(w^*) = 0$. $S(w^*) < \hat{S}$. This implies that for some world prices, opening up to trade decreases social welfare .*

Proof. Utility maximization implies that if the price ratio is w, the ratio of amounts consumed is $x(w)$. The tax scheme induces a production ratio equal to $\pi_s l_s(w)/\pi_u l_u(w)$. The country's net export of the unskilled good, $t_u(w)$, has therefore the same sign as

$$x(w) - \frac{\pi_s l_s(w)}{\pi_u l_u(w)},$$

which is an increasing function of w. If the world price ratio is $w(\varepsilon)$, this expression is equal to

$$x(w(\varepsilon)) - \frac{\pi_s l_s}{\pi_u l_u}(w(\varepsilon)) = \varepsilon - \frac{\pi_s l_s}{\pi_u l_u}(w(\varepsilon)).$$

As ε converges toward zero, $\pi_s l_s(w(\varepsilon))/\pi_u l_u(w(\varepsilon))$ does not, being a decreasing function of ε. Therefore, if ε is small enough, $t_u(\varepsilon) < 0$: if the price of the unskilled good is small enough, it is optimal to import it and to export the skilled good. We know from Proposition 2 that $t_u(\hat{w}) > 0$, so that by continuity there exists $w^* < \hat{w}$ such that $t_u(w^*) = 0$.

The fact that there is no trade when the world price ratio is w^* and the pair of labor supplies is $(l_u(w^*), l_s(w^*))$ implies that the consumption ratio is equal to the production ratio, so that $w^* = w\left(\pi_s l_s(w^*)/\pi_u l_u(w^*)\right)$. Therefore the welfare level $S(w^*)$ can be attained in autarky by assigning the pair of labor supplies $(l_u(w^*), l_s(w^*))$:

$$\begin{aligned} S_{open}\left(l_u(w^*), l_s(w^*), w^*\right) &= S_{open}\left(l_u(w^*), l_s(w^*), w\left(\pi_s l_s(w^*)/\pi_u l_u(w^*)\right)\right) \\ &= S_{closed}\left(l_u(w^*), l_s(w^*)\right). \end{aligned}$$

But the inequalities $w\left(\pi_s l_s(w^*)/\pi_u l_u(w^*)\right) = w^* < \hat{w} = w\left(\pi_s \hat{l}_s/\pi_u \hat{l}_s\right)$ imply that $\left(\hat{l}_s, \hat{l}_u\right) \neq (l_u(w^*), l_s(w^*))$, so that

$$S(w^*) = S_{open}\left(l_u(w^*), l_s(w^*), w^*\right) = S_{closed}\left(l_u(w^*), l_s(w^*)\right) < S_{closed}\left(\hat{l}_s, \hat{l}_u\right) = \hat{S}.$$

■

This result answers our main question: opening up to trade may decrease social welfare. It is remarkable that the equilibrium with no trade but open borders is inferior to the autarkic optimum. The reason is that although no trade takes place, the opening of borders has an effect: the government is not able any more to change the wage ratio through the general equilibrium effects of taxation, and this increases wage inequality. Equally surprising perhaps, opening up to trade may decrease welfare even if it causes the country to export the unskilled good. Indeed, Proposition 4 implies by continuity that $S(w) < \hat{S}$ for w slightly above w^*, but $w > w^*$ implies that $t_u(w) > 0$. Again, this shows that the immediate intuition may be misleading: the adverse effect

of open borders on unskilled wages need not occur through actual imports of unskilled labor intensive goods. The channel is rather through the sudden impossibility for the government to affect wages. This inability removes the general equilibrium motivations for distorting unskilled labor supply downwards (and skilled labor supply upwards). This may increase the production of the unskilled good relative to that of the skilled good, and therefore cause the country to export the unskilled labor intensive good.

The following result completes the description of the function $S(w)$.

Proposition 5 *Assume that* $\underset{w \to 0}{Lim}\ w_s(w) = \infty$. *Then* $S(w)$ *is U-shaped, reaches its minimum at* $w_{\min} < w^* < \hat{w}$, *and* $S(w) > \hat{S}$ *if* w *is small enough (see Figure 1).*

Proof. See the appendix. ■

Notice first that the condition $\underset{w \to 0}{Lim}\ w_s(w) = \infty$ is satisfied if the elasticity of substitution between the two goods is constant and belongs to the interval $(0, 1)$. The intuition behind this result is the following: "extreme" values of w (very low or very high) greatly expand the set of feasible outcomes of the country, and the gains from trade are so large that it is possible to compensate the losers: the "terms of trade" effect dominates the redistributive effect. In the case of intermediate prices on the contrary, the country is deprived of the possibility to manipulate equilibrium wages, without reaping large gains from trade. More specifically, if $\underset{w \to 0}{Lim}\ w_s(w) = \infty$, then the wage of skilled labor tends to infinity as w tends to zero, so that utility per capita tends to infinity for any given pair of labor supplies, and even a small redistribution to the unskilled is enough to make them better off than in autarky.

The inequality $w_{\min} < w^*$ has the following implication: in the interval $[w_{\min}, w^*]$, a decrease in w improves the terms of trade (because the country is importing the unskilled good if $w < w^*$) but decreases social welfare. If the country were inhabited by homegeneous agents, or if lump-sum transfers were feasible, then welfare would be minimal when world prices induce no trade. Once constraints on redistribution are taken into account, this ceases to be true, and an improvement in the terms of trade at the expense of low wages may decrease welfare.

6 Two examples

The following examples, which depart from the model analyzed so far, may clarify the logic of Proposition 4. What may cause opening up to trade to decrease social welfare is the fact that if borders are open, the demand for each good becomes infinitely elastic, depriving the government of the possibility of affecting equilibrium wages through the tax scheme.

6.1 Endogenizing world prices: the case of many identical countries

Proposition 6 *Assume that there are N identical countries (N large), characterized by π_s and π_u, and the utility function is given by (1). Then welfare (measured by the utility of the unskilled) is larger if trade is forbidden than if borders are open, although open borders result in no trade in equilibrium.*

Proof. Welfare in autarky is $\hat{S}$. If borders are open, then there is no trade in equilibrium because countries are identical. Since N is large, each government takes wages as being exogenous and optimizes its fiscal policy given the wage ratio w, leading to the choice of $(l_u(w), l_s(w))$. For no trade to take place, it must be the case that the equilibrium world price ratio is w^*, and we saw in Proposition 4 that $S(w^*) < \hat{S}$. Therefore welfare is lower if borders are open than if trade is forbidden. ■

This result shows that the possibility for income taxation to manipulate equilibrium wages generates an externality if borders are open. The tax scheme prevailing in autarky remains of course feasible, but if it were prevailing in all countries, it would be optimal for any single country to deviate: given wages, setting labor supplies maximizing (6) (taking the wage ratio $\hat{w}$ as exogenous), would increase the country's welfare. But this deviation imposes a negative externality on other countries: it increases the supply of unskilled labor and decreases the supply of skilled labor, increasing therefore wage inequality in the whole world. In other words, a country fails to recognize that when it enacts a policy decreasing wage inequality at home, it also decreases it abroad. Notice that this argument is not related to any mobility of the tax base.

6.2 The case of an infinite elasticity of substitution

In this subsection, we modify the model to show that if the elasticity of the demand for each type of good is infinite even when borders are closed (because the utility function displays an infinite elasticity of substitution), then opening up to trade unambiguously raises welfare after the tax scheme adjusts. In other words, if direct control of prices was already impossible in the closed economy, then the argument made in the previous example does not apply any more, and welfare increases[13].

Proposition 7 *Assume that the utility function is $U(C_s, C_u, l) = w_s^0 C_s + w_u^0 C_u - V(l)$. Then if the economy opens up to trade, the function $S(w)$ is U-shaped, and reaches its minimum at $w^0 = w_u^0 / w_s^0$, where social welfare is the same as in autarky (see Figure 2).*

Proof. Notice first that the function $w(x)$ is constant so that the first-order conditions are the same in a closed economy and in an open economy where the price ratio is w^0, so that $S(w^0) = \hat{S}$. Consider a price ratio w. The utility function implies that the inhabitants of the country consume only the unskilled (resp. skilled) good if $w < w^0$ (resp. $w > w^0$). As in the previous sections, let $(l_u(w), l_s(w))$ denote the optimal pair of labor supplies chosen by the government when the world price ratio is w. The same calculation as in Section 2 shows that

$$S(w) = \begin{cases} w_u^0 \left(\pi_u l_u(w) + \pi_s w^{-1} l_s(w)\right) + V(w l_u(w)) - \pi_s V(l_s(w)) - V(l_u(w)) \text{ if } w \leq w^0 \\ w_s^0 \left(\pi_u w l_u(w) + \pi_s l_s(w)\right) + V(w l_u(w)) - \pi_s V(l_s(w)) - V(l_u(w)) \text{ if } w \geq w^0. \end{cases}$$

It is clear, by the envelope theorem, that S is increasing over $[w^0, 1]$. If $w < w^0$, then the envelope theorem again implies that

$$\frac{1}{\pi_s}\frac{\partial S}{\partial w} = -\frac{w_u^0}{w^2} l_s(w) + l_u(w) V'(w l_u(w))$$

[13] The case investigated in this subsection is the closest one to Mirrlees [1971]: as in his model, we assume wages to be exogenous even in a closed economy. The only difference between his model and ours is that he considers a world with a unique good (and an infinite elasticity of substitution in production), while we consider a world with two goods and an infinite elasticity of substitution in consumption. The two models are formally equivalent in a closed economy, but our formulation also allows to model an opening up of the economy, which would be impossible if there were only one good - and therefore no trade.

The inequality $w < 1$ and the fact that, at the fiscal optimum, $l_s(w) > l_u(w)$ and $V'(wl_u(w)) < V'(l_u(w)) < w_u(w) < w_u^0$ implies that the above derivative is negative. Together with the continuity of S, this implies that S is U-shaped, and reaches its minimum at $w = w^0$, at which point welfare equals welfare in autarky. ∎

Proposition 7 implies that our results break down if we are in the world studied by Mirrlees [1971]: the decrease in the unskilled wage caused by the move from autarky to free trade is not enough, in itself, to decrease unskilled workers' utility. Proposition 6 showed that for trade to decrease welfare, it is enough that it weakens the government's ability to affect wages. Proposition 7 shows that it is also necessary: if trade just changes relative wages while keeping the ability to change equilibrium wages at its initial level (that is, assuming it is *already* impossible in a closed economy), then it is always beneficial provided the tax scheme is properly modified.

Notice also that in the case considered in this subsection, Naito's result still holds, since a tariff on imports (if $w \leq w^0$) or a tax on exports (if $w \geq w^0$) would increase welfare further: for a given pair (l_u, l_s), a small increase in w does not affect the amount of trade (as long as the sign of $w - w^0$ remains the same) and therefore the average consumption of the single good consumed does not change. However an increase in w makes unskilled workers better off. This remark should make clear that Naito's point and ours are different: he compares the optimum with versus without tariffs in any given open economy, while we compare the optimum without tariffs in an open economy versus in autarky.

7 Concluding remarks

Several conclusions can be drawn from the simple model developed in this paper. First, many of the usual intuitions about the effects of trade are wrong if one takes fiscal policy into account. In particular, it appears from Figure 1 than *whatever the prevailing world prices*, opening up to trade changes something: it is impossible that both wages and quantities remain at their autarky level. In other words, there is a lower bound on how "small" the effect of trade can be. This implies that the frequent analogy between trade and technical progress (relying on the idea that opening up to trade amounts to adding an "exchange technology" to the set of available techniques) may be misleading: technical change can have arbitrarily small effects, trade cannot.

The reason is that demand elasticities "jump" discontinuously to infinity if a small country opens up to trade.

Second, this jump of labor demand elasticities implies that it may be impossible for the government to redistribute the gains from trade in a Pareto-improving manner. The infinite elasticity of demand prevents the government from equalizing wages by manipulating labor supply through the tax system, as it did in autarky. The interpretation of this effect is a little awkward, because our modeling of the optimal redistributive policy in the case of endogenous wages (the same as in Stiglitz [1982]) should not be interpreted literally: instead of increasing low wages indirectly by a reduction of the labor supply of the unskilled caused by high marginal tax rates, as Stiglitz's model predicts, governments often resort to more direct policies, such as imposing a minimum wage or restricting hours worked. However, the mechanism highlighted in this model can easily be interpreted in terms of these real-world institutions: the increase in labor demand elasticity caused by the opening of borders reduces the effectiveness of such institutions.

This mechanism relies strongly on the assumption that the goods produced by unskilled workers in various countries are perfect substitutes, and on the fact that we are considering a small country taking world prices as given (otherwise labor demand elasticities would not become infinite). These two assumptions seem more appropriate in the case of north-north trade than in the case of north-south trade: there is obviously more substitutability between unskilled labor in Germany and in the United States than between unskilled labor in Chad and in Sweden; also, the North being a much bigger economy than the South, it is more likely that the South is taking prices as given, than the opposite. Therefore the question of the impact of trade on inequality should not restrict its attention to North-South trade. Trade between similar countries may also have significant effects.

How robust is the argument? It should be clear that for our point to be valid, it is not necessary that elasticities jump to infinity when a country opens up to trade: it is enough that they increase. This has two implications: first, the small country assumption is not essential. Second, our results carry over to the case where the production function for each good allows for some substitution between labor types. In the presence of some substitutability in production, opening up causes the elasticity of the demand for goods to become infinite, but this is not true any more of the elasticity of the demand for each type of labor: the government can still increase the unskilled wage by decreasing its supply, that is, by encouraging firms to substitute high-

skilled labor for low-skilled labor. However, it is still true that the "indirect" elasticities of labor demand increase discontinuously, since they are functions both of production elasticities and of the elasticities of the demand for goods.

Also, the result stating that opening up to trade may decrease welfare should not be taken literally. We left unmodeled, indeed, the main aspect of north-north trade, which has to do with intra-industry specialization, and is likely to generate important efficiency gains. In other words, when identical countries form a common market, the negative result stated in Proposition 6 is incomplete: contrary to our analysis, there is trade in equilibrium, and the loss of some redistributive tools is mitigated by the efficiency gain brought about by trade, making the total effect on welfare ambiguous.

However, we believe the main point of this paper to have some relevance even if trade actually increases welfare. The fact that international trade weakens the effectiveness of institutions such as minimum wages, or of other attempts to redistribute income by interfering with markets (for example through trade union pressure), suggests that direct income redistribution through the tax system tends to become the only tool by which open economies can meet demands for inequality reduction.

8 Appendix

Proof of Lemma 1.

The first-order conditions[14] associated to the maximization of (6) are

$$w_s(w) - V'(l_s) = 0 \tag{8}$$

and

$$\pi_u w_u(w) - V'(l_u) + \pi_s w V'(l_u w) = 0, \tag{9}$$

The function $SW_{open}(l_s, l_u, w)$, to maximize, can be written as the sum of

$$\pi_s \left[w_s(w) l_s - V(l_s) \right],$$

which is a strictly concave function of l_s, and of

$$\pi_u l_u w_u(w) + \pi_s V(l_u w) - V(l_u).$$

The second derivative of this latter function with respect to l_u is

$$g(l_u, w) = \pi_s w^2 V''(l_u w) - V''(l_u).$$

Clearly $g(l_u, 1) < 0$, and

$$\partial g / \partial w(l_u, w) = w \left(2V''(l_u w) + V'''(l_u w) \right).$$

Assumption (2) implies that for any l, $2V''(l)+V'''(l) \geq 0$, so that $\frac{\partial g}{\partial w}(l_u, w) \geq 0$. Therefore for any $w \leq 1$, $g(l_u, w) \leq g(l_u, 1) < 0$, so that $[w_u(w) l_u + \pi_s V(l_u w) - V(l_u)]$ is a strictly concave function of l_u. Therefore $SW_{open}(l_s, l_u, w)$ is a strictly concave function of (l_s, l_u), which implies that (6) has a unique solution $(l_u(w), l_s(w))$, continuous in w. The first order condition (8) implies that $l_s(w)$ increases strictly with $w_s(w)$, that is, decreases strictly with w. Similarly, the left-hand side of (9), $\pi_u w_u(w) - V'(l_u) + \pi_s w V'(l_u w)$, is a strictly decreasing function of l_u (its derivative with respect to l_u is $g(l_u, w)$, which was shown above to be negative because of assumption (2), and it is a strictly increasing function of w, so that $l_u(w)$ is strictly increasing.

[14]The assumptions $V'(0) = 0$ and $V'(\infty) = \infty$ ensure that the solution is interior and satisfies the first-order conditions.

Proof of Lemma 3. F being homogeneous of degree one and $(x_s(w), x_u(w))$ being an optimal consumption pair when prices are $(w_s(w), w_u(w))$ implies the identity

$$F(x_s(w), x_u(w)) = w_u(w)x_u(w) + w_s(w)x_s(w).$$

Therefore $S(w)$ is given by

$$\begin{aligned}
S(w) &= \pi_u w_u(w) l_u(w) + \pi_s w_s(w) l_s(w) - \pi_s \left(V(l_s(w)) - V(w l_u(w))\right) - V(l_u(w)) \\
&= w_u(w)x_u(w) + w_s(w)x_s(w) - \pi_s \left(V(l_s(w)) - V(w l_u(w))\right) - V(l_u(w)) \\
&= F(x_s(w), x_u(w)) - \pi_s \left(V(l_s(w)) - V(w l_u(w))\right) - V(l_u(w))
\end{aligned}$$

Balanced trade implies that $x_s(w) = w\left(\pi_u l_u(w) - x_u(w)\right) + \pi_s l_s(w)$. Therefore the above identity can be written as

$$S(w) = Z(w, x_u(w), l_s(w), l_u(w))$$

with

$$Z(w, x, l_s, l_u) = F\left(w\left(\pi_u l_u - x_u\right) + \pi_s l_s, x_u\right) - \pi_s \left(V(l_s) - V(w l_u)\right) - V(l_u)$$

The envelope theorem and the first-order condition $\partial F/\partial x_s = w_s$ imply that

$$\begin{aligned}
\frac{dS}{dw} &= \frac{\partial Z}{\partial w} = \left(\pi_u l_u - x_u\right)\frac{\partial F}{\partial x_s} + \pi_s l_u(w) V'(l_u w) \\
&= w_s(w) t_u(w) + \pi_s l_u(w) V'(l_u w)
\end{aligned}$$

which proves the result.

Proof of Proposition 5.

Lemma 3 and Proposition 4 imply that if $w \geq w^*$, then $S'(w) > 0$: an increase in w is an improvement of the terms of trade since the country is exporting the unskilled good if $w \geq w^*$, so that the "terms of trade" effect and the direct redistributive effect go in the same direction. We claim now that $S(w)$ is a convex function of w in the interval $(0, w^*)$. In lemma 3 we established that

$$S'(w) = t_u(w) w_s(w) + \pi_s l_u(w) V'(l_u w).$$

Lemma 1 implies that l_u increases with w. This and the convexity of V implies that the second term in the expression above, $\pi_s l_u(w) V'(l_u w)$, is an increasing function of w. By Proposition 4, we know that $t_u(w) < 0$ if $w \in (0, w^*)$. Therefore, in order to show that the first term in the derivative above, $t_u(w) w_s(w)$, increases with w, we need to show that it decreases in absolute value. We rewrite this term as $t_u(w) w_s(w) = \frac{w_s(w)}{w} w t_u(w)$. Since $\frac{w_s(w)}{w}$ is a positive and decreasing function of w, it is enough to show that $w t_u(w)$ is an increasing function on $(0, w^*)$.

Consider a pair w, w' such that $w < w'$, and assume that $w t_u(w) > w' t_u(w')$, which implies that $t_u(w) > t_u(w')$. Simple accounting and the monotonicity properties of the functions $l_u(w)$ and $l_s(w)$ yield

$$\begin{cases} x_s(w') = \pi_s l_s(w') + w' t_u(w') < \pi_s l_s(w) + w t_u(w) = x_s(w) \\ x_u(w') = \pi_u l_u(w') - t_u(w') > \pi_u l_u(w) - t_u(w) = x_u(w) \end{cases}$$

so that

$$x(w') = \frac{x_s(w')}{x_u(w')} < \frac{x_s(w)}{x_u(w)} = x(w)$$

which is impossible because $x(w)$ is an increasing function. This proves that $S(w)$ is a convex function of w in the interval $(0, w^*)$.

If we pick an arbitrary l_0, then for any w

$$\begin{aligned} S(w) &= \pi_u w_u(w) l_u + \pi_s w_s(w) l_s - \pi_s \left(V(l_s) - V(l_u w)\right) - V(l_u) \\ &\geq \pi_u w_u(w) l_0 + \pi_s w_s(w) l_0 - \pi_s \left(V(l_0) - V(l_0 w)\right) - V(l_0) \\ &\geq \left(\pi_u w_u(w) + \pi_s w_s(w)\right) l_0 - (1 + \pi_s) V(l_0) + \pi_s V(0). \end{aligned}$$

The assumption $\underset{w \to 0}{Lim}\, w_s(w) = \infty$ implies that $\underset{w \to 0}{Lim}\, S(w) = \infty$. This together with the fact that $S'(w^*) > 0$ and the convexity of S on $(0, w^*)$ implies that S is U-shaped on $(0, w^*)$ and reaches a minimum for some $w_{\min} < w^*$. That S is increasing on $(w^*, 1)$ implies finally that S is U-shaped on $(0, 1)$.

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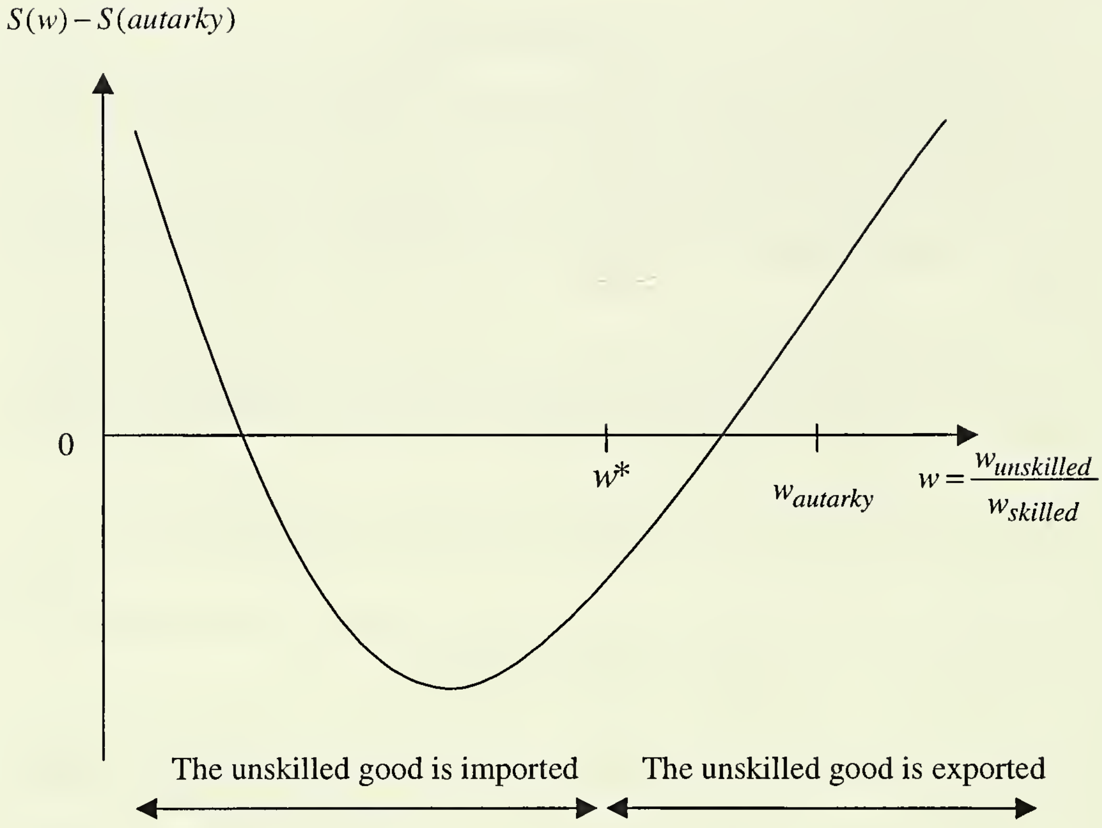

Figure 1:

Impact of trade on unskilled workers' utility if the government maximizes it using a nonlinear income tax and the elasticity of substitution between both goods is finite.

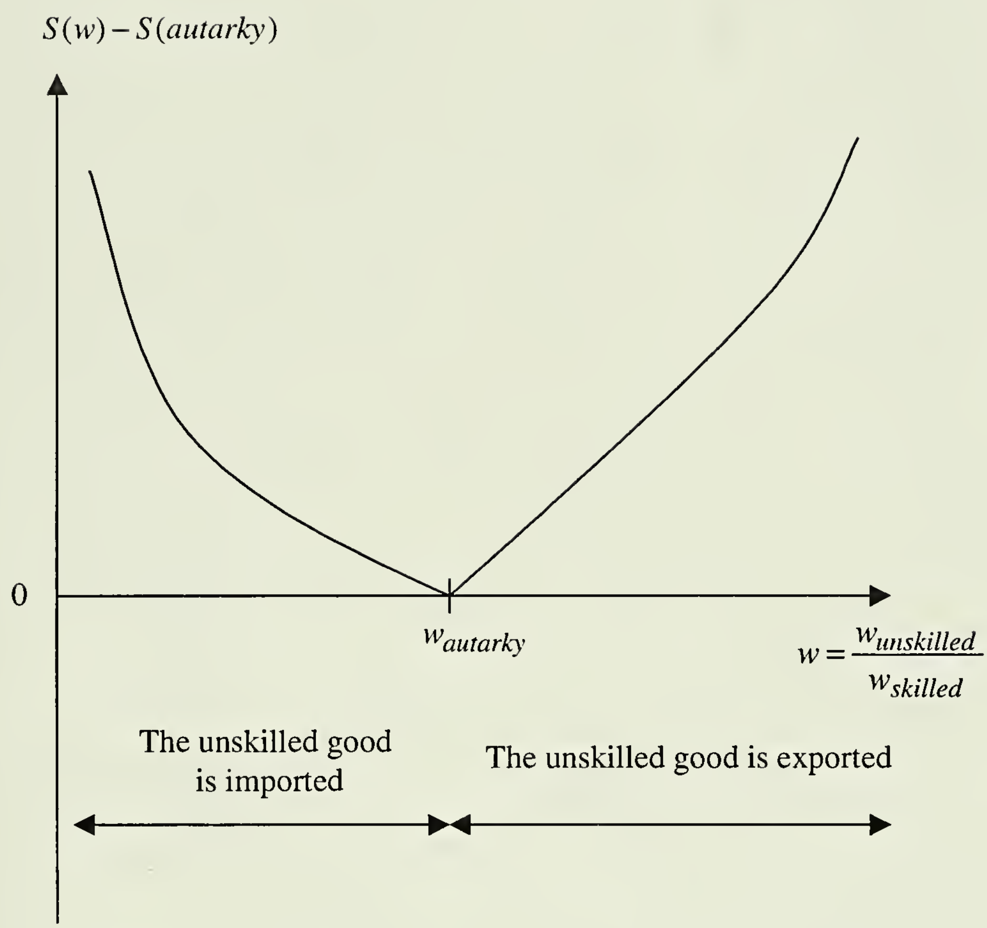

Figure 2:

Impact of trade on unskilled workers' utility if the government maximizes it using a nonlinear income tax and the elasticity of substitution between both goods is infinite.

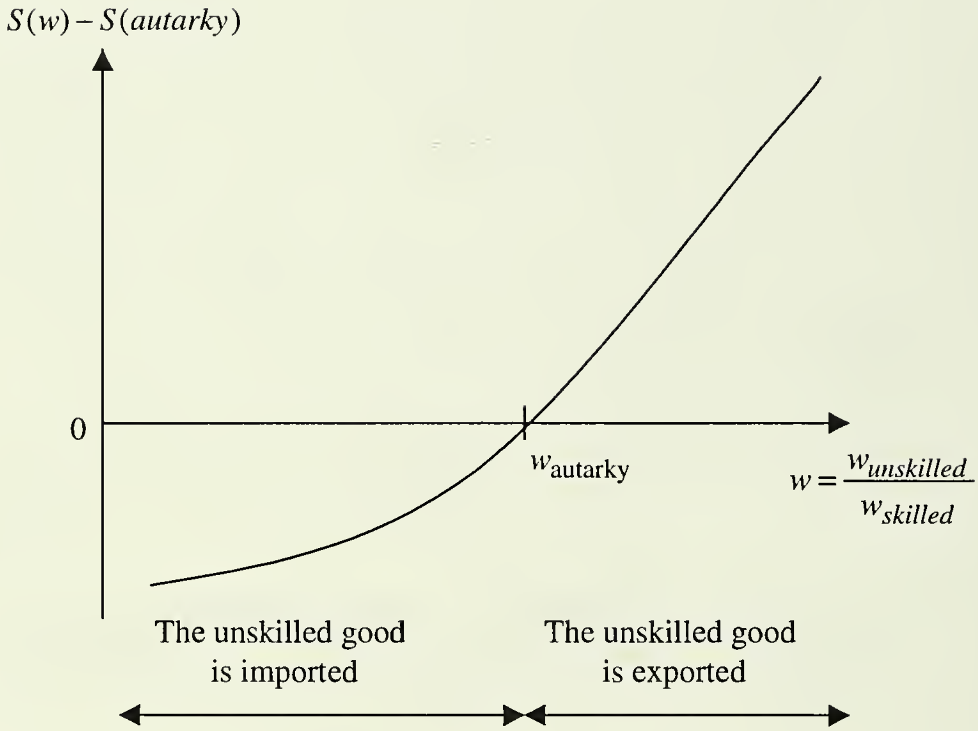

Figure 3:

Impact of trade on unskilled workers' utility if there is no government